

Dibaryons with two heavy quarks

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The relativistic six-quark equations are constructed in the framework of the dispersion relation technique. The relativistic six-quark amplitudes of dibaryons including the light u , d and heavy c , b quarks are calculated. The approximate solutions of these equations using the method based on the extraction of leading singularities of the heavy hexaquark amplitudes are obtained. The poles of these amplitudes determine the masses of charmed and bottom dibaryons with the isospins $I = 0$, 1 , 2 and the spin-parities $J^P = 0^+$, 1^+ , 2^+ .

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I. INTRODUCTION.

In 1977, Jaffe [1] studied the color-magnetic interaction of the one-gluon-exchange potential in the multiquark system and found that the most attractive channel is the flavor singlet with quark content $u^2 d^2 s^2$. The same symmetry analysis of the chiral boson exchange potential leads to the similar result [2].

The H -particle, $N\Omega$ -state and di- Ω may be strong interaction stable. Up to now, these three interesting candidates of dibaryons are still not found or confirmed by experiments. It seems that one should go beyond these candidates and should search for the possible candidates in a wider region, especially the systems with heavy quarks, in terms of a more reliable model.

There were a number of theoretical predictions by using various models: the quark cluster model [3, 4], the quark-delocation model [5, 6], the chiral $SU(3)$ quark model [7], the flavor $SU(3)$ skyrmion model [8]. Lomon predicted a deuteronlike dibaryon resonance using R-matrix theory [9]. By employing the chiral $SU(3)$ quark model Zhang and Yu studied $\Omega\Omega$ and $\Sigma\Omega$ states [10, 11]. Lee and Yasui discuss the stable multiquark states containing charm and bottom quark [12].

In a series of papers [13–17] a method has been developed which is convenient for analyzing relativistic three-hadron systems. The physics of the three-hadron system can be described by means of a pair interaction between the particles. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and of analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In our papers [18–20] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The mass spectrum of S -wave baryons including u , d , s quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, and defined all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the present paper the relativistic six-quark equations are found in the framework of coupled-channel formalism. We use only planar diagrams; the other diagrams due to the rules of $1/N_c$ expansion [21–23] are neglected. The six-quark amplitudes of dibaryons with two heavy quarks are calculated. The poles of these amplitudes determine the masses of heavy dibaryons.

In Sec. II, the six-quark amplitudes of hexaquarks are constructed. The dynamical mixing between the subamplitudes of dibaryons are considered. The relativistic six-quark equations are constructed in the form of the dispersion relation over the two-body subenergy. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. Sec. III is devoted to the calculation results for the dibaryon mass spectra (Tables I, II). In conclusion, the status of the considered model is discussed.

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II. SIX-QUARK AMPLITUDES OF THE HEXAQUARKS WITH THE TWO HEAVY QUARKS.

We derive the relativistic six-quark equations in the framework of the dispersion relation technique. We use only planar diagrams; the other diagrams due to the rules of $1/N_c$ expansion [21–23] are neglected. The current generates a six-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. Then one should represent a six-particle amplitude as a sum of 15 subamplitudes:

$$A = \sum_{\substack{i < j \\ i, j=1}}^6 A_{ij}. \quad (1)$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. We need to consider only one group of diagrams and the amplitude corresponding to them, for example A_{12} . We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach. In our case, the low-lying dibaryons with the two heavy quarks are considered. We take into account the pairwise interaction of all six quarks in the hexaquark.

For instance, we consider the state $\Sigma_c \Sigma_c$ with the isospin $I = 2$ and the spin-parity $J^P = 0^+$ ($uuc\ uuc$). The set of diagrams associated with the amplitude A_{12} can further be broken down into seven groups corresponding to subamplitudes: A_1^{1uu} , A_1^{1cc} , A_1^{0uc} , A_2^{1uu1uu} , A_2^{1uu0uc} , A_2^{0uc0uc} , $A_3^{1uu1uu1cc}$.

The system of graphical equations (see for example equation for the amplitude A_2^{0uc0uc} for the state $\Sigma_c \Sigma_c$ with the isospin $I = 2$ and the spin-parity $J^P = 0^+$ ($uuc\ uuc$)) is determined by the subamplitudes using the self-consistent method. The coefficients are determined by the permutation of quarks.

In order to represent the subamplitudes A_1^{1uu} , A_1^{1cc} , A_1^{0uc} , A_2^{1uu1uu} , A_2^{1uu0uc} , A_2^{0uc0uc} , $A_3^{1uu1uu1cc}$ in the form of a dispersion relation, it is necessary to define the amplitude of qq , qQ and QQ interactions. We use the results of our relativistic quark model [24] and write down the pair quark amplitudes in the form:

$$a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \quad (2)$$

$$B_n(s_{ik}) = \frac{(m_i + m_k)^2 \Lambda}{\int_{(m_i + m_k)^2}^4} \frac{ds'_{ik}}{\pi} \frac{\rho_n(s'_{ik}) G_n^2(s'_{ik})}{s'_{ik} - s_{ik}}, \quad (3)$$

$$\begin{aligned} \rho_n(s_{ik}, J^P) &= \left(\alpha(n, J^P) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(n, J^P) + \delta(n, J^P) \frac{(m_i - m_k)^2}{s_{ik}} \right) \\ &\times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \end{aligned} \quad (4)$$

The coefficients $\alpha(n, J^P)$, $\beta(n, J^P)$ and $\delta(n, J^P)$ are given in Table III. Here $n = 1$ corresponds to qq , qQ and QQ -pairs with $J^P = 0^+$, $n = 2$ corresponds to qq , qQ and QQ with $J^P = 1^+$.

The coupled integral equations correspond to Fig. 1 can be described similar to [25].

Then we can go from the integration of the cosine of the angles dz_i to the integration over the subenergies.

Let us extract two- and three-particle singularities in the amplitudes A_1^{1uu} , A_1^{1cc} , A_1^{0uc} , A_2^{1uu1uu} , A_2^{1uu0uc} , A_2^{0uc0uc} , $A_3^{1uu1uu1cc}$:

$$A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{1uu}(s_{12})}{[1 - B_{1uu}(s_{12})]}, \quad (5)$$

$$A_1^{1cc}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1cc}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{1cc}(s_{12})}{[1 - B_{1cc}(s_{12})]}, \quad (6)$$

$$A_1^{0uc}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{0uc}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) B_{0uc}(s_{12})}{[1 - B_{0uc}(s_{12})]}, \quad (7)$$

$$A_2^{1^{uu}1^{uu}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}1^{uu}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})B_{1^{uu}}(s_{12})B_{1^{uu}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{1^{uu}}(s_{34})]}, \quad (8)$$

$$A_2^{1^{uu}0^{uc}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1^{uu}0^{uc}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})B_{1^{uu}}(s_{12})B_{0^{uc}}(s_{34})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{0^{uc}}(s_{34})]}, \quad (9)$$

$$A_2^{0^{uc}0^{uc}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{0^{uc}0^{uc}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})B_{0^{uc}}(s_{12})B_{0^{uc}}(s_{34})}{[1 - B_{0^{uc}}(s_{12})][1 - B_{0^{uc}}(s_{34})]}, \quad (10)$$

$$A_3^{1^{uu}1^{uu}1^{cc}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) = \frac{\alpha_3^{1^{uu}1^{uu}1^{cc}}(s, s_{12345}, s_{12}, s_{34}, s_{56})B_{1^{uu}}(s_{12})B_{1^{uu}}(s_{34})B_{1^{cc}}(s_{56})}{[1 - B_{1^{uu}}(s_{12})][1 - B_{1^{uu}}(s_{34})][1 - B_{1^{cc}}(s_{56})]}. \quad (11)$$

We used the classification of singularities, which was proposed in paper [26]. Using this classification, one defines the reduced amplitudes α_1 , α_2 , α_3 as well as the B -functions in the middle point of physical region of Dalitz-plot at the point s_0 .

Such choice of point s_0 allows us to replace integral equations ($\Sigma_c \Sigma_c$, $I = 2$, $J^P = 0^+$) by the algebraic equations (12) – (18):

$$\begin{aligned} \alpha_1^{1^{uu}} &= \lambda + 4\alpha_1^{1^{uu}}I_1(1^{uu}1^{uu}) + 4\alpha_1^{0^{uc}}I_1(1^{uu}0^{uc}) + 2\alpha_2^{1^{uu}1^{uu}}I_2(1^{uu}1^{uu}1^{uu}) + 8\alpha_2^{1^{uu}0^{uc}}I_2(1^{uu}1^{uu}0^{uc}) \\ &+ 2\alpha_2^{0^{uc}0^{uc}}I_2(1^{uu}0^{uc}0^{uc}) \end{aligned} \quad (12)$$

$$\alpha_1^{1^{cc}} = \lambda + 8\alpha_1^{0^{uc}}I_1(1^{ss}0^{uc}) + 12\alpha_2^{0^{uc}0^{uc}}I_2(1^{cc}0^{uc}0^{uc}) \quad (13)$$

$$\begin{aligned} \alpha_1^{0^{uc}} &= \lambda + 3\alpha_1^{1^{uu}}I_1(0^{uc}1^{uu}) + \alpha_1^{1^{cc}}I_1(0^{uc}1^{cc}) + 4\alpha_1^{0^{uc}}I_1(0^{uc}0^{uc}) + 6\alpha_2^{1^{uu}0^{uc}}I_2(0^{uc}1^{uu}0^{uc}) \\ &+ 3\alpha_2^{0^{uc}0^{uc}}I_2(0^{uc}0^{uc}0^{uc}) \end{aligned} \quad (14)$$

$$\begin{aligned} \alpha_2^{1^{uu}1^{uu}} &= \lambda + 4\alpha_1^{1^{uu}}I_3(1^{uu}1^{uu}1^{uu}) + 8\alpha_1^{0^{uc}}I_4(1^{uu}1^{uu}0^{uc}) + 16\alpha_2^{1^{uu}0^{uc}}I_7(1^{uu}1^{uu}1^{uu}0^{uc}) \\ &+ \alpha_2^{0^{uc}0^{uc}}(4I_5(1^{uu}1^{uu}0^{us}0^{uc}) + 8I_6(1^{uu}1^{uu}0^{uc}0^{uc})) \end{aligned} \quad (15)$$

$$\begin{aligned} \alpha_2^{1^{uu}0^{uc}} &= \lambda + \alpha_1^{1^{uu}}(2I_3(1^{uu}0^{uc}1^{uu}) + I_4(0^{uc}1^{uu}1^{uu})) + \alpha_1^{0^{uc}}(2I_3(1^{uu}0^{uc}0^{uc}) + 2I_4(1^{uu}0^{uc}0^{uc})) \\ &+ \alpha_2^{1^{uu}0^{uc}}(2I_5(1^{uu}0^{uc}1^{uu}0^{uc}) + 2I_6(1^{uu}0^{us}0^{uc}1^{uu}) + 2I_7(1^{uu}0^{uc}1^{uu}0^{uc}) + 2I_7(1^{uu}0^{us}0^{uc}1^{uu}) \\ &+ 2I_7(0^{uc}1^{uu}1^{uu}0^{uc})) + \alpha_2^{0^{uc}0^{uc}}(I_5(0^{uc}1^{uu}0^{uc}0^{uc}) + 2I_6(1^{uu}0^{uc}0^{uc}0^{uc}) + 2I_7(0^{uc}1^{uu}0^{uc}0^{uc})) \end{aligned} \quad (16)$$

$$\begin{aligned} \alpha_2^{0^{uc}0^{uc}} &= \lambda + \alpha_1^{1^{uu}}(I_3(0^{uc}0^{uc}1^{uu}) + 4I_4(0^{uc}0^{uc}1^{uu})) + \alpha_1^{1^{cc}}I_3(0^{uc}0^{uc}1^{cc}) + \alpha_1^{0^{uc}}(2I_3(0^{uc}0^{uc}0^{uc}) \\ &+ 4I_4(0^{uc}0^{uc}0^{uc})) + 2\alpha_2^{1^{uu}1^{uu}}I_6(0^{uc}0^{uc}1^{uu}1^{uu}) + \alpha_2^{1^{uu}0^{uc}}(4I_5(0^{uc}0^{uc}1^{uu}0^{uc}) \\ &+ 4I_6(0^{uc}0^{uc}1^{uu}0^{uc}) + 4I_7(0^{uc}0^{uc}1^{uu}0^{uc}) + 4I_7(0^{uc}0^{uc}0^{uc}1^{uu})) + \alpha_2^{0^{uc}0^{uc}}(2I_6(0^{uc}0^{uc}0^{uc}0^{uc}) \\ &+ 4I_7(0^{uc}0^{uc}0^{uc}0^{uc})) + 2\alpha_3^{1^{uu}1^{uu}1^{cc}}I_8(0^{uc}0^{uc}1^{uu}1^{cc}1^{uu}) \end{aligned} \quad (17)$$

$$\alpha_3^{1^{uu}1^{uu}1^{cc}} = \lambda + 4\alpha_1^{1^{uu}}I_9(1^{uu}1^{uu}1^{cc}1^{uu}) + 8\alpha_1^{0^{uc}}I_9(1^{uu}1^{cc}1^{uu}0^{uc}) + 16\alpha_2^{1^{uu}0^{uc}}I_{10}(1^{uu}1^{uu}1^{cc}1^{uu}0^{uc})$$

$$+ 8 \alpha_2^{0^{uc}0^{uc}} I_{10}(1^{uu}1^{cc}1^{uu}0^{uc}0^{uc}), \quad (18)$$

where λ_i are the current constants. We used the functions $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}$ similar to the paper [27]. The solutions of the system of equations are considered as:

$$\alpha_i(s) = \frac{F_i(s, \lambda_i)}{D(s)}, \quad (19)$$

where zeros of $D(s)$ determinants define the masses of bound states of dibaryons.

III. CALCULATION RESULTS.

The model in question take into account the hexaquarks with the two heavy quarks $qqqqQQ$, $q = u, d$, $Q = c, b$: $uuuucc$, $uuudcc$, $uuddcc$, $uuuubb$, $uuudbb$, $uuddbb$.

The quark masses of the model are $m_q = 495 \text{ MeV}$, $m_c = 1655 \text{ MeV}$ and $m_b = 4840 \text{ MeV}$.

The experimental data are absent, therefore we use the dimensionless parameters, which are similar to the previous paper [25]. It allows us to calculate the mass spectra of $qqqqQQ$ states. We use the gluon coupling constants $g_0 = 0.653$ (diquark $J^P = 0^+$) and $g_1 = 0.292$ (diquark $J^P = 1^+$), cutoff parameter $\Lambda = 11$. We consider the $\Lambda_{qc,cc} = 8.52$, which are determined by $M = 5250 \text{ MeV}$ ($IJ = 22 \Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^*$), the threshold is 5290 MeV . In the case of b -quarks the $\Lambda_{qb,bb} = 7.35$ is determined by $M = 11620 \text{ MeV}$ ($IJ = 22 \Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^*$), the threshold is 11660 MeV .

We have calculated the heavy dibaryon masses with isospin $I = 0, 1, 2$ and spin-parity $J^P = 0^+, 1^+, 2^+$, which are given in the Tables I and II. The relativistic six-body approach possesses the dynamical mixing and allows us to calculate the contributions of the subamplitudes to the hexaquark amplitude (Tables IV – VI). The calculated dibaryon subamplitudes A_2 present the main contributions to the hexaquark amplitude (about 70 percents). We use only two new parameters for the calculation of 23 $qqqqcc$ states and 19 $qqqqbb$ states.

The lowest mass for the $qqqqcc$ states is $M = 4364 \text{ MeV}$, for the $qqqqbb$ states is $M = 8670 \text{ MeV}$.

In quark models, which describe rather well the masses and static properties of hadrons, the masses of the quarks usually have the similar values for the spectra of light and heavy hadrons. However, this is achieved at the expense of some difference in the characteristic of the confinement potential. It should be borne in mind that for a fixed hadron mass the masses of the constituent quarks which enter into the composition of the hadron will become smaller when the slope of the confinement potential increases or its radius decreases. Therefore, conversely, we can change the masses of the constituent quarks when going from the spectrum of light to the heavy hadrons, while keeping the characteristic of the confinement potential unchanged. We can effectively take into account the contribution of the confinement potential in obtaining the spectrum of heavy hadrons. We neglect with the mass distinction of u and d quarks. The estimation of the theoretical error on the heavy dibaryons masses is 1 MeV . This result was obtained by the choice of model parameters.

IV. CONCLUSIONS.

In a strongly bound systems, which include the light quarks, where $p/m \sim 1$, the approximation of nonrelativistic kinematics and dynamics is not justified. In our paper, the relativistic description of six-particles amplitudes of heavy dibaryons with the two heavy quarks is considered. We take into account the u, d, c, b quarks. Our model is confined to the quark-antiquark pair production on account of the phase space restriction. Here m_q and m_Q the "mass" of the constituent quark. Therefore the production of new quark-antiquark pair is absent for the low-lying hadrons.

Hadronic molecules are loosely states of hadrons, whose inter-hadron distances are larger than the quark confinement size.

The heavy analogue of H dibaryon ($\Lambda_c \Lambda_c$) does not exist though its potential is attractive [28]. Oka et al. believe that the future studies may specify the binding energy of such a molecule state. The binding energy is sensitive to the cutoff parameter [29]. Our calculation allows us to obtain the $\Lambda_c \Lambda_c$ molecule bound state (Table I). There exist a loosely bound state with a small binding energy $E_B = 54 \text{ MeV}$. In the case of $\Sigma_c \Sigma_c$ dibaryon we obtain the bound state, but the binding energy is equal to $E_B = 540 \text{ MeV}$.

The similar results are obtained for the other heavy dibaryons with the two heavy quarks (Tables I – II). For the $\Lambda_b \Lambda_b$ system does not exist a loosely bound state with a small binding energy. The heavy dibaryons and heavy baryon-antibaryons may be produced at LHC.

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TABLE I: S-wave charmed dibaryon masses. Parameters of model: cutoff $\Lambda = 11.0$ and $\Lambda_{qc,cc} = 8.52$, gluon coupling constants $g_0 = 0.653$ and $g_1 = 0.292$. Quark masses $m_q = 495 \text{ MeV}$ and $m_c = 1655 \text{ MeV}$.

I	J	Dibaryons (quark content)	Mass (MeV)
2	0	$\Sigma_c \Sigma_c, \Sigma_c^* \Sigma_c^* (uuc\ uuc)$	4933
		$\Delta \Xi_{cc}^* (uuu\ ucc)$	5231
	1	$\Sigma_c \Sigma_c, \Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^* (uuc\ uuc)$	4933
		$\Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uuu\ ucc)$	5231
	2	$\Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^* (uuc\ uuc)$	5250
		$\Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uuu\ ucc)$	5231
1	0	$\Sigma_c \Sigma_c, \Sigma_c^* \Sigma_c^*, \Sigma_c \Lambda_c (uuc\ udc)$	4420
		$\Delta \Xi_{cc}^* (uuu\ dcc + uud\ ucc)$	4956
		$N \Xi_{cc} (uud\ ucc)$	4956
	1	$\Sigma_c \Sigma_c, \Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^*, \Sigma_c \Lambda_c, \Sigma_c^* \Lambda_c (uuc\ udc)$	4420
		$\Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uuu\ dcc + uud\ ucc)$	4956
		$N \Xi_{cc}, N \Xi_{cc}^* (uud\ ucc)$	4956
	2	$\Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^*, \Sigma_c^* \Lambda_c (uuc\ udc)$	4911
		$\Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uuu\ dcc + uud\ ucc)$	4999
		$N \Xi_{cc}^* (uud\ ucc)$	5136
	0	$\Sigma_c \Sigma_c, \Sigma_c^* \Sigma_c^*, (uuc\ ddc + udc\ udc)$	4364
		$\Sigma_c \Lambda_c, \Lambda_c \Lambda_c (udc\ udc)$	4516
		$N \Xi_{cc}, \Delta \Xi_{cc}^* (uud\ dcc + udd\ ucc)$	4740
0	1	$\Sigma_c \Sigma_c, \Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^*, (uuc\ ddc + udc\ udc)$	4364
		$\Sigma_c \Lambda_c, \Sigma_c^* \Lambda_c \Lambda_c \Lambda_c (udc\ udc)$	4516
		$N \Xi_{cc}, N \Xi_{cc}^*, \Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uud\ dcc + udd\ ucc)$	4740
	2	$\Sigma_c \Sigma_c^*, \Sigma_c^* \Sigma_c^*, (uuc\ ddc + udc\ udc)$	5086
		$N \Xi_{cc}^*, \Delta \Xi_{cc}, \Delta \Xi_{cc}^* (uud\ dcc + udd\ ucc)$	5029

TABLE II: S-wave bottom dibaryon masses. Parameters of model: cutoff $\Lambda = 11.0$ and $\Lambda_{qb,bb} = 7.35$, gluon coupling constants $g_0 = 0.653$ and $g_1 = 0.292$. Quark masses $m_q = 495 \text{ MeV}$ and $m_b = 4840 \text{ MeV}$.

I	J	Dibaryons (quark content)	Mass (MeV)
2	0	$\Sigma_b \Sigma_b, \Sigma_b^* \Sigma_b^* (uub uub)$	10290
		$\Delta \Xi_{bb}^* (uuu ubb)$	—
	1	$\Sigma_b \Sigma_b, \Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^* (uub uub)$	10290
		$\Delta \Xi_{bb}, \Delta \Xi_{bb}^* (uuu ubb)$	—
	2	$\Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^* (uub uub)$	11620
		$\Delta \Xi_{bb}, \Delta \Xi_{bb}^* (uuu ubb)$	—
1	0	$\Sigma_b \Sigma_b, \Sigma_b^* \Sigma_b^*, \Sigma_b \Lambda_b (uub udb)$	8670
		$\Delta \Xi_{bb}^* (uuu dbb + uud ubb)$	11395
		$N \Xi_{bb} (uud ubb)$	11395
	1	$\Sigma_b \Sigma_b, \Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^*, \Sigma_b \Lambda_b, \Sigma_b^* \Lambda_b (uub udb)$	8670
		$\Delta \Xi_b, \Delta \Xi_{bb}^* (uuu dbb + uud ubb)$	11395
		$N \Xi_{bb}, N \Xi_b^* (uud ubb)$	11395
	2	$\Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^*, \Sigma_b^* \Lambda_b (uub udb)$	10715
		$\Delta \Xi_{bb}, \Delta \Xi_{bb}^* (uuu dbb + uud ubb)$	11372
		$N \Xi_{bb}^* (uud ubb)$	—
	0	$\Sigma_b \Sigma_b, \Sigma_b^* \Sigma_b^*, (uub ddb + udb udb)$	8482
		$\Sigma_b \Lambda_b, \Lambda_b \Lambda_b (udb udb)$	9175
		$N \Xi_{bb}, \Delta \Xi_{bb}^* (uud dbb + udd ubb)$	10828
0	1	$\Sigma_b \Sigma_b, \Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^*, (uub ddb + udb udb)$	8482
		$\Sigma_b \Lambda_b, \Sigma_b^* \Lambda_b \Lambda_b \Lambda_b (udb udb)$	9175
	2	$N \Xi_{bb}, N \Xi_{bb}^*, \Delta \Xi_{bb}, \Delta \Xi_{bb}^* (uud dbb + udd ubb)$	10828
		$\Sigma_b \Sigma_b^*, \Sigma_b^* \Sigma_b^*, (uub ddb + udb udb)$	11518
	2	$N \Xi_{bb}^*, \Delta \Xi_{bb}, \Delta \Xi_{bb}^* (uud dbb + udd ubb)$	11583

TABLE III: Vertex functions and Chew-Mandelstam coefficients.

i	$G_i^2(s_{kl})$	α_i	β_i	δ_i
0^+	$\frac{4g}{3} - \frac{8gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
1^+	$\frac{2g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$

TABLE IV: $IJ = 00$ $\Sigma_Q \Sigma_Q$, $\Sigma_Q^* \Sigma_Q^*$. Parameters of model: cutoff $\Lambda = 11.0$, $\Lambda_{qc,cc} = 8.52$, $\Lambda_{qb,bb} = 7.35$, gluon coupling constants $g_0 = 0.653$ and $g_1 = 0.292$. Quark masses $m_q = 495 \text{ MeV}$, $m_c = 1655 \text{ MeV}$ and $m_b = 4840 \text{ MeV}$.

Subamplitudes	Contributions, percent	
	$Q = c$	$Q = b$
A_1^{1uu}	2.6	4.6
A_1^{1dd}	2.6	4.6
A_1^{1QQ}	3.7	6.4
A_1^{0ud}	6.4	10.2
A_1^{0uQ}	3.1	2.5
A_1^{0dQ}	3.1	2.5
$A_3^{1uu1dd1QQ}$	2.6	1.8
$A_3^{0ud0uQ0dQ}$	2.0	0.2
A_2^{1uu1dd}	6.1	10.2
A_2^{1uu0dQ}	2.8	2.2
A_2^{1dd0uQ}	2.8	2.2
A_2^{0ud0ud}	25.4	35.5
A_2^{0ud0uQ}	7.4	5.0
A_2^{0ud0dQ}	7.4	5.0
A_2^{0uQ0uQ}	7.8	2.6
A_2^{0dQ0dQ}	7.8	2.6
A_2^{0uQ0dQ}	6.6	2.2
$\sum A_1$	21.4	30.6
$\sum A_2$	74.0	67.5
$\sum A_3$	4.6	2.0

TABLE V: $IJ = 00 \Sigma_Q \Lambda_Q, \Lambda_Q \Lambda_Q$. Parameters of model: cutoff $\Lambda = 11.0$, $\Lambda_{qc,cc} = 8.52$, $\Lambda_{qb,bb} = 7.35$, gluon coupling constants $g_0 = 0.653$ and $g_1 = 0.292$. Quark masses $m_q = 495 \text{ MeV}$, $m_c = 1655 \text{ MeV}$ and $m_b = 4840 \text{ MeV}$.

Subamplitudes	Contributions, percent	
	$Q = c$	$Q = b$
A_1^{1uu}	3.0	5.6
A_1^{1dd}	3.0	5.6
A_1^{1QQ}	4.3	8.3
A_1^{0ud}	7.0	11.4
A_1^{0uQ}	3.6	3.2
A_1^{0dQ}	3.6	3.2
$A_3^{0ud0uQ0dQ}$	3.1	0.3
A_2^{0udd}	28.8	39.8
A_2^{0ud0uQ}	9.2	6.9
A_2^{0ud0dQ}	9.2	6.9
A_2^{0uQ0uQ}	10.1	3.6
A_2^{0dQ0dQ}	10.1	3.6
A_2^{0uQ0dQ}	5.1	1.8
$\sum A_1$	24.5	37.2
$\sum A_2$	72.4	62.5
$\sum A_3$	3.1	0.3

TABLE VI: $IJ = 00 N\Xi_{QQ}, \Delta\Xi_{QQ}^*$. Parameters of model: cutoff $\Lambda = 11.0$, $\Lambda_{qc,cc} = 8.52$, $\Lambda_{qb,bb} = 7.35$, gluon coupling constants $g_0 = 0.653$ and $g_1 = 0.292$. Quark masses $m_q = 495 \text{ MeV}$, $m_c = 1655 \text{ MeV}$ and $m_b = 4840 \text{ MeV}$.

Subamplitudes	Contributions, percent	
	$Q = c$	$Q = b$
A_1^{1uu}	2.1	1.8
A_1^{1dd}	2.1	1.8
A_1^{1QQ}	3.6	5.7
A_1^{0ud}	6.6	6.2
A_1^{0uQ}	3.7	2.1
A_1^{0dQ}	3.7	2.1
$A_3^{1uu1dd1QQ}$	4.9	4.8
$A_3^{0ud0uQ0dQ}$	4.9	0.6
A_2^{1uu1QQ}	10.6	15.7
A_2^{1dd1QQ}	10.6	15.7
A_2^{1uu0dQ}	3.0	10.0
A_2^{1dd0uQ}	3.0	10.0
A_2^{1QQ0ud}	22.3	34.6
A_2^{0ud0uQ}	9.5	3.6
A_2^{0ud0dQ}	9.5	3.6
$\sum A_1$	21.7	19.6
$\sum A_2$	68.6	75.1
$\sum A_3$	9.8	5.4

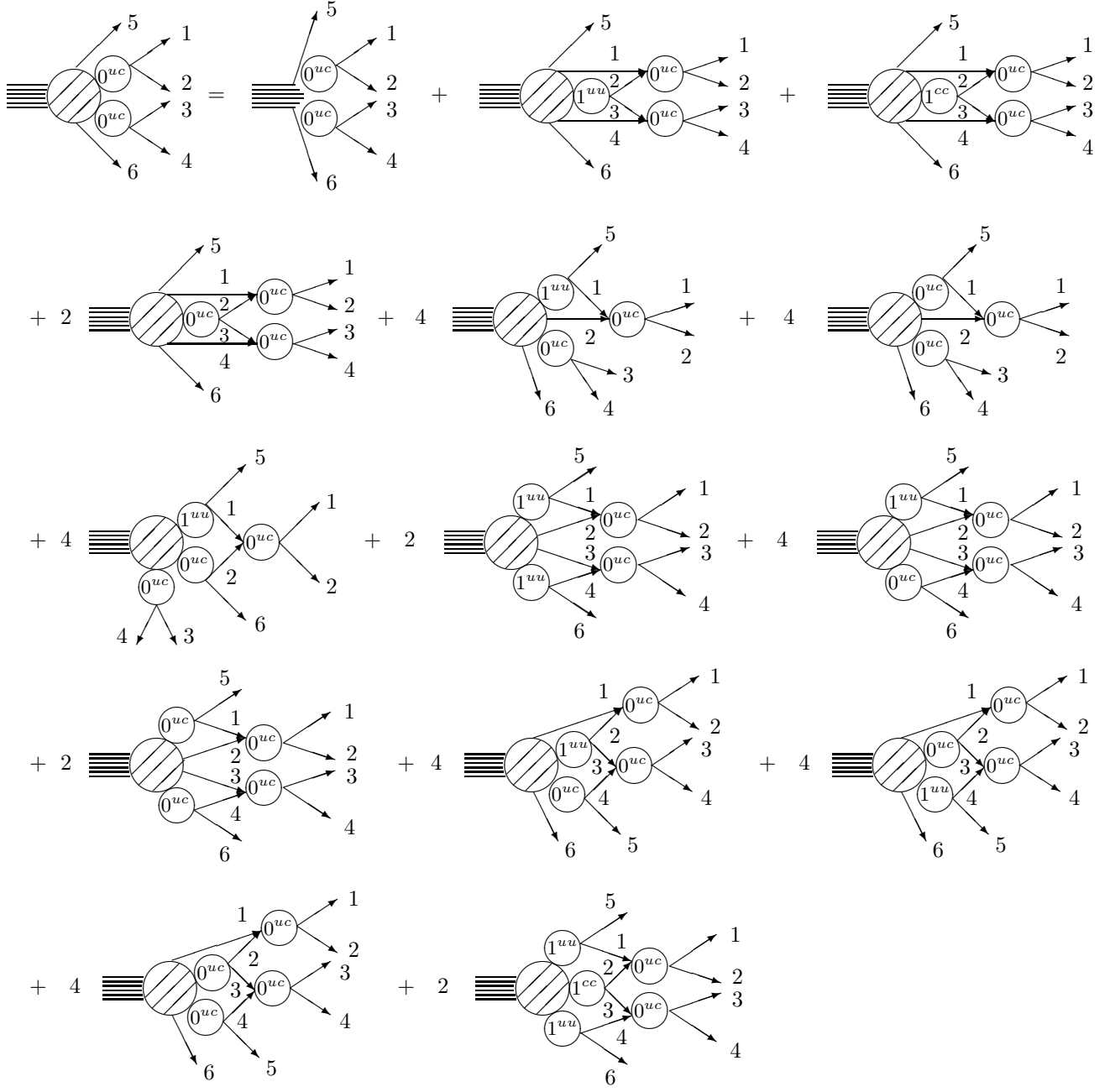


Fig. 1. Graphical representation of the amplitude $A_2^{0^{uc}0^{uc}}$ for the case of $\Sigma_c \Sigma_c, \Sigma_c^* \Sigma_c^*, I=2, J^P=0^+$.